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Equivalent linear stochastic seismic response of isolated bridges

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Abstract

Stochastic response of bridges seismically isolated by lead-rubber bearings (LRB) is investigated. The earthquake excitation is modeled as a non-stationary random process (i.e. uniformly modulated broad-band excitation). The stochastic response of isolated bridge is obtained using the time-dependent equivalent linearization technique as the force-deformation behavior of the LRB is highly nonlinear. The non-stationary response of isolated bridge is compared with the corresponding stationary response in order to study the effects of non-stationary characteristics of the earthquake input motion. For a given isolated bridge system and excitation, it was observed that there exists an optimum value of the yield strength of LRB for which the root mean square (rms) absolute acceleration of bridge deck attains the minimum value. The optimum yield strength of LRB is investigated under important parametric variations such as isolation period and damping ratio of the LRB and the frequency content and intensity of earthquake excitation. It is shown that the above parameters have significant effects on the optimum yield strength of LRB. Finally, closed-form expressions for the optimum yield strength of LRB and corresponding response of the isolated bridge system are proposed. These expressions were derived based on the model of bridge with rigid deck and pier condition subjected to stationary white-noise excitation. It was observed that there is a very good comparison between the proposed closed-form expressions and actual optimum parameters and response of the isolated bridge system. These expressions can be used for initial optimal design of seismic isolation system for the bridges.

1. Introduction

Seismic isolation has emerged as one of the most promising technique for retrofitting strategies to improve the seismic performance of existing bridges. It is also an attractive approach for new construction when conventional design is not suitable or economical. In the seismic isolation approach, the superstructure mass is uncoupled from seismic ground motions. It uses special types of bearings known as seismic isolation bearings which are placed below the superstructure and on top of the substructure (piers and/or abutments). Under normal conditions, these bearings behave like conventional bearings. However, in the event of a strong earthquake, they add flexibility to the bridge by elongating its period and dissipate the input energy. This permits the superstructure to oscillate at a lower frequency than that of the piers. It could also give rise to large relative displacements across the isolator interface, which can be controlled by incorporating damping elements in the bearing or by adding supplemental dampers. Seismic isolation provides two significant design

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features for a bridge namely (i) it can reduce the seismic forces substantially and (ii) it can control the distribution of these reduced lateral forces among the substructures and foundations to further enhance the overall economy and effectiveness of new and retrofit designs. The details of earlier research and recent investigations on seismic isolation of bridges can be found in review Ref. [1]. The design guidelines and specifications for bridges with seismic isolators to take advantage of the reduced earthquake forces and provide the most economical bridge design with same level of seismic safety are also available in the AASHTO code [2].

There have been several studies in the past for investigating the effectiveness of isolation devices for aseismic design of bridges [3–14]. Ghobarah and Ali [3] and Turkington et al. [4] showed that the lead-rubber bearings (LRB) are quite effective in reducing the seismic response of bridges. Hwang and Sheng [5] evaluated specified effective stiffness and equivalent damping ratio for an equivalent elastic system of isolated bridges and proposed empirical formulae for the calculations of the equivalent period shift and equivalent damping ratio. Jangid [6] studied the bi-directional response of bridges isolated by LRB and showed that the bi-directional interaction of bearing restoring forces had considerable effects on the seismic response of bridges. Constantinou et al. [7] and Tsopelas et al. [8] conducted experimental studies on bridges isolated by sliding bearings with displacement control devices to real earthquake ground motion applied independently in longitudinal and transverse direction. Kartoum et al. [9] and Wang et al. [10] studied the effectiveness of friction pendulum system for seismic isolation of bridges. Li [11] and Pagnini and Solari [12] presented the response of bridges isolated by the LRB to stationary earthquake ground motion using equivalent linearization technique. Recently, Ates et al. [13,14] investigated the stochastic response of seismically isolated highway bridges with friction pendulum systems to spatially varying earthquake ground motion. The review of above studies indicates that there had not been much attempt to investigate the response of isolated bridges to non-stationary earthquake excitations.

Here in, the stochastic response of multi-span continuous deck bridge isolated by the LRB to non-stationary earthquake ground motion is presented. The specific objectives of the study are to: (i) present mathematical formulations for evaluation of response of isolated bridge subjected non-stationary earthquake ground motion, (ii) study the effects of yield strength of LRB on the stochastic response of isolated bridge system, (iii) investigate the existence of optimum yield strength of the LRB for a given structural system and excitation, (iv) study the effects of important system parameters on the variation of optimum yield strength of the LRB. The parameters included are isolation period and damping ratio of the LRB and the frequency and intensity of earthquake excitation, and (v) to present approximate closed-form expressions for the optimum yield strength of LRB and corresponding response of the isolated bridge system for design purposes.

2. Modeling of isolated bridge system

Fig. 1(a) shows the general elevation of a bridge consisting multi-span continuous deck supported by the isolation system. The substructure of bridge consists of rigid abutments and reinforced concrete piers. For the present study, the LRB consisting of alternating layers of steel shims and rubber with central lead plug are considered as isolation device. The LRB is very stiff in vertical direction and flexible in horizontal direction (due to presence of steel shims and rubber). The horizontal flexibility and damping characteristics of the LRB provides desired isolation effects in the system [15,16]. The horizontal flexibility transmits relatively limited earthquake forces from substructures to the superstructure. On the other hand, the damping of bearings dissipates the seismic energy thereby reducing the design displacement of the bridge. In addition, the inelastic deformation of lead plug also provides the hysteretic damping in the system. The following assumptions are made for the earthquake analysis of isolated bridge under consideration:

1. The deck of bridge is straight and is supported at discrete locations along its longitudinal axis by cross diaphragms and the angle of skew is zero. The bridge deck is idealized as a rigid body.
2. Bridge piers are assumed to remain in the elastic state during the earthquake excitation. This is a reasonable assumption as the isolation attempts to reduce the earthquake response in such a way that the structure remains within the elastic range.

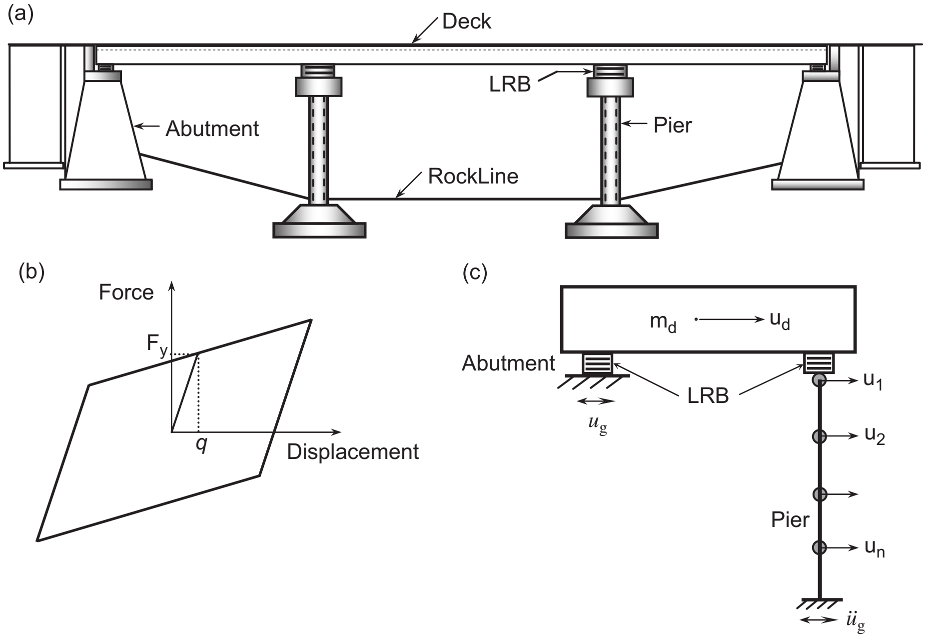

Fig. 1. General elevation and mathematical modeling of the bridge and isolation devices.

3. The piers are modeled as lumped mass system divided into number of small discrete segments. Each adjacent segment is connected by a node and at each node single degree-of-freedom is considered. The masses of each segment are assumed to be distributed between the two adjacent nodes in the form of point masses.
4. Bridge piers are assumed to be rigidly fixed at the foundation level.
5. The LRB is considered to be isotropic implying the same dynamic properties in two orthogonal directions. In addition, the bearings provided at the piers and abutments have the same dynamic characteristics.
6. The force-deformation behavior of the LRB is considered as bi-linear (refer Fig. 1(b)) based on the nonlinear hysteretic model.

The above assumptions will lead to the mathematical model of the isolated bridge system as shown in Fig. 1(c) which was studied earlier in Refs. [3,9,11,17]. The governing equations of motion of the isolated bridge model are expressed by

$$m_d\ddot{u}_d + F_a + F_p = -m_d\ddot{u}_g, \tag{1}$$

$$[m_p]\{\ddot{u}_p\} + [c_p]\{\dot{u}_p\} + [k_p]\{u_p\} - \{\psi\}F_p = -[m_p]\{1\}\ddot{u}_g, \tag{2}$$

where m_d is the mass of the bridge deck; $[m_p]$, $[c_p]$ and $[k_p]$ represents the mass, stiffness and damping matrix of size $n \times n$, respectively, of the pier under top free condition; n is the number of nodes considered in the pier; F_a and F_p represent the restoring forces of the LRB at abutment and pier level, respectively; u_d is the displacement of the deck relative to ground; $\{u_p\} = \{u_1, u_2, \ldots, u_n\}^{\mathrm{T}}$ is the vector of the displacement of various nodes of the pier; u_i is the horizontal displacement of the ith node of the pier; $\{\psi\} = \{1, 0, \ldots, 0\}^{\mathrm{T}}$ is a vector of size $n \times 1$; $\{1\} = \{1, 1, \ldots, 1\}^{\mathrm{T}}$ is the influence coefficient vector of size $n \times 1$; and $\ddot{u}_g$ represents the earthquake ground acceleration.

The damping matrix of the pier is not explicitly known and it is constructed from assumed modal damping (ξ_p) in each mode of vibration using its mode-shapes and frequencies. For the present study, the number of nodes considered in the pier is five (i.e. $n = 5$). The damping in the pier is taken as 2 percent of the critical in all modes of vibration. The pier of the bridge model is considered to be of uniform cross-section throughout the height. The fundamental time period of the pier, T_p with top free condition is expressed as

$$T_p = \frac{2\pi}{(1.875)^2}\sqrt{\frac{\bar{m}_p h^4}{EI}}, \tag{3}$$

where $\bar{m}_p$ is the mass per unit length of the pier; h is the height of the pier; and EI is the flexural rigidity of the pier. Note that Eq. (3) is based on the fundamental time period of a uniform cantilever beam under transverse vibrations.

The ratio of pier mass to the deck mass is expressed by μ defined as

$$\mu = \frac{\bar{m}_p h}{m_d}. \tag{4}$$

3. Mathematical model of LRB

For the present study, the bi-linear force-deformation behavior of the LRB is expressed by the Wen's equation [18]. The restoring force of the LRB is expressed by

$$F_j = c_b \dot{u}_j + \alpha k_b u_j + (1-\alpha) F_y Z_j \qquad (\text{for } j = a \text{ and } p), \tag{5}$$

where c_b represent the viscous damping of the LRB or the damping provided by the additional viscous dampers; k_b is the initial stiffness of the LRB; u_j and $\dot{u}_j$ represents the relative displacement and velocity in the bearing, respectively (i.e. $u_a = u_d$ and $u_p = u_d - u_1$); u_1 is the displacement of the pier top (refer in Fig. 1(c)); α is an index, which represents the ratio of post to pre-yielding stiffness of the LRB; F_y is the yield strength of the bearing; and Z_j is a non-dimensional hysteretic component satisfying the following nonlinear first-order differential equation expressed as

$$q\dot{Z}_j = A\dot{u}_j - \gamma|\dot{u}_j|Z_j|Z_j|^{\eta-1} - \beta\dot{u}_j|Z_j|^{\eta} \qquad (\text{for } j = a \text{ and } p), \tag{6}$$

where q is the yield displacement of the bearing; β, γ η and A are non-dimensional parameters of the hysteresis loop. The parameters α, β, γ, A and η control the shape of the loop and are selected such that the predicted response from the model closely matches with the experimental results [19]. The parameters considered for the present study are: $\alpha = 0.05$, $\beta = \gamma = 0.5$, $A = 1$ and $\eta = 1$ taken from Refs. [12,19].

The LRB for isolation of the bridges are designed to provide the specified values of three parameters namely the T_b, ξ_b and F_0 expressed as

$$T_b = 2\pi\sqrt{\frac{m_d}{\sum \alpha k_b}}, \tag{7}$$

$$\xi_b = \frac{\sum c_b}{2 m_d \omega_b}, \tag{8}$$

$$F_0 = \frac{\sum F_y}{W_d}, \tag{9}$$

where T_b is the isolation time period of the bearings; $\sum \alpha k_b$ is the total post-yield stiffness of the bearings; ξ_b is the damping ratio; $\sum c_b$ is the total viscous damping of bearings or added viscous dampers; $\omega_b = 2\pi/T_b$ is the isolation frequency; F_0 is the normalized yield strength of LRB; $\sum F_y$ is the total yield strength of LRB; $W_d = m_d\, g$ is the deck weight; and g is the acceleration due to gravity.

4. Model of earthquake excitation

Earthquake ground motions are inherently random and multidimensional. If the evolution of the frequency content with time can be neglected, the ground motion can be characterized by a matrix of the power spectral density function (PSDF) and an intensity envelope function. The earthquake excitation is considered as a uniformly modulated stationary Gaussian random process with zero-mean. The earthquake acceleration, $\ddot{u}_g(t)$ is expressed as

$$\ddot{u}_g(t) = A(t)\ddot{u}_f(t), \tag{10}$$

where $A(t)$ is the deterministic modulating function; and $\ddot{u}_f(t)$ is the stationary random process. The deterministic modulating function $A(t)$ in Eq. (10) is considered from Ref. [20] expressed as

$$A(t) = \begin{cases} (t/t_1)^2 & (0 \leqslant t \leqslant t_1), \\ 1 & (t_1 \leqslant t \leqslant t_2), \\ \mathrm{e}^{-c(t-t_2)} & (t \geqslant t_2), \end{cases} \tag{11}$$

where t_1 and t_2 denotes the time for start and end of strong motion duration, respectively; c is a constant; and $T_0 = t_2 – t_1$ is known as the strong motion duration of earthquake.

The evolutionary PSDF of the earthquake excitation is given as

$$S_{\ddot{u}_g}(\omega) = |A(t)|^2 S_{\ddot{u}_f}(\omega), \tag{12}$$

where $S_{\ddot{u}_f}(\omega)$ is the stationary PSDF of the earthquake ground motion. In the present study, the PSDF of the earthquake excitation, $\ddot{u}_f(t)$ is considered as that suggested by Clough and Penzien [21], i.e.

$$S_{\ddot{u}_f}(\omega) = S_0\left(\frac{1 + 4\xi_g^2(\omega/\omega_g)^2}{[1 - (\omega/\omega_g)^2]^2 + 4\xi_g^2(\omega/\omega_g)^2}\right)\left(\frac{(\omega/\omega_f)^4}{[1 - (\omega/\omega_f)^2]^2 + 4\xi_f^2(\omega/\omega_f)^2}\right), \tag{13}$$

where S_0 is the constant PSDF of input white-noise random process; and ω_g, ξ_g, ω_f and ξ_f are the ground filter parameters.

The ω_g and ξ_g generally represent the pre-dominant frequency and damping ratio of the soil strata, respectively. For the present study, the values of various parameters considered are: $\xi_g = \xi_f = 0.6$ and $\omega_f = 0.1\omega_g$. The parameters ωg and S_0 are varied to study the influence of pre-dominant earthquake frequency and intensity of excitation on the response of the isolated bridge system.

Note that the state variable method for stochastic response of any system requires that the excitation must be either white-noise or shot-noise whereas the PSDF of $\ddot{u}_f(t)$ is a non-white random process. However, this obstacle can be circumvented by introducing the shaping filters in which the random process, $\ddot{u}_f(t)$ can be considered as the response of two linear filters subjected to white-noise excitation as

$$\ddot{u}_f(t) + 2\xi_f\omega_f\dot{u}_f(t) + \omega_f^2 u_f(t) = -(\ddot{u}_g(t) + \ddot{u}_0(t)), \tag{14}$$

$$\ddot{u}_g(t) + 2\xi_g\omega_g\dot{u}_g(t) + \omega_g^2 u_g(t) = -\ddot{u}_0(t), \tag{15}$$

where $\ddot{u}_0(t)$ is the input white-noise random process with constant intensity of the PSDF as S_0. Note that Eqs. (14) and (15) provide the stationary PSDF of the response, $\ddot{u}_f(t)$ as that expressed by the Eq. (13).

5. Evaluation of stochastic response

The Eq. (6) representing the force-deformation characteristics of LRB is nonlinear equation, which cannot be used when the response is to be evaluated using state variable approach. Thus, Eq. (6) is replaced by an equivalent linear equation [22] as

$$q\dot{Z}_j + C_j\dot{u}_j + K_jZ_j = 0, \tag{16}$$

where C_j and K_j are the equivalent constants which are obtained by minimizing the mean square error between the linear and nonlinear terms. For $\eta = 1$, the equivalent constants C_j and K_j are given by

$$C_j = \sqrt{\frac{2}{\pi}}\left\{\gamma\frac{E[\dot{u}_j, Z_j]}{\sqrt{E[\dot{u}_j, \dot{u}_j]}} + \beta\sqrt{E[Z_j, Z_j]}\right\} - A, \tag{17}$$

$$K_j = \sqrt{\frac{2}{\pi}}\left\{\gamma\sqrt{E[\dot{u}_j,\dot{u}_j]} + \beta\frac{E[\dot{u}_j,Z_j]}{\sqrt{E[Z_j,Z_j]}}\right\}, \tag{18}$$

where $E[]$ is the expectation operator.

Eqs. (1,2) and (5,6) along with Eqs. (14) and (15) can be re-written as a system of first-order differential equations as

$$\frac{\mathrm{d}}{\mathrm{d}t}\{Y\} = [H]\{Y\} + \{W\}, \tag{19}$$

where $\{Y\}$ is the state vector, $[H]$ is the augmented system matrix, and $\{W\}$ is the excitation vector.

The corresponding $[H]$ matrix is expressed in Appendix I for the isolated bridge model considered in the present study. The vector $\{Y\}$ and $\{W\}$ are expressed as

$$\{Y\} = \left\{ u_d \quad \{u_p\}^{\mathrm{T}} \quad \dot{u}_d \quad \{\dot{u}_p\}^{\mathrm{T}} \quad Z_a \quad Z_p \quad u_f \quad \dot{u}_f \quad u_g \quad \dot{u}_g \right\}^{\mathrm{T}}, \tag{20}$$

$$\{W\} = \left\{ 0 \quad \{0\} \quad 0 \quad \{0\} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -\ddot{u}_0 \right\}^{\mathrm{T}}. \tag{21}$$

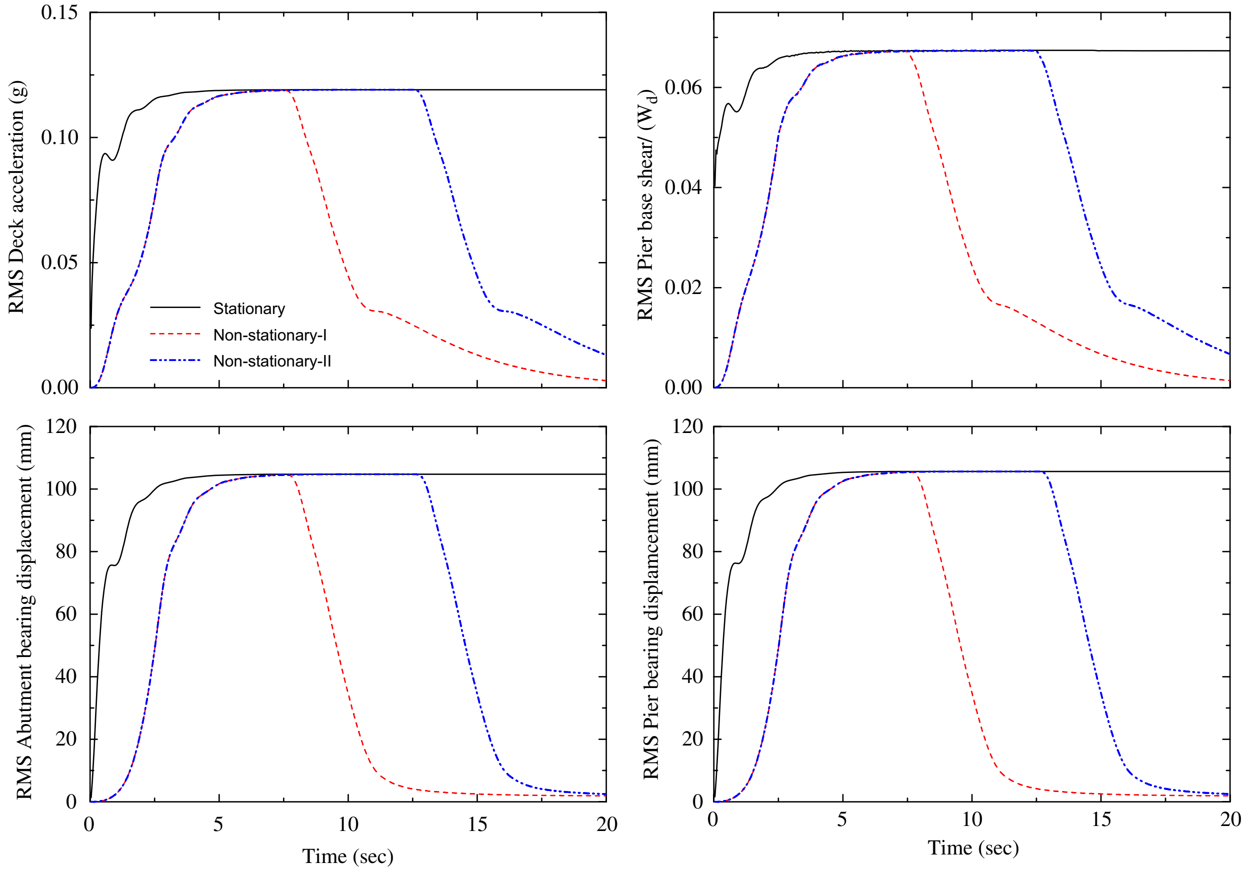

Fig. 2. Comparison of the stationary and non-stationary response of seismically isolated bridge ($T_p = 0.05$ s, $\mu = 0.15$, $T_b = 2$ s, $\xi_b = 0.1$, $F_0 = 0.05$, $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m}^2/\mathrm{s}^3$).

The augmented response vector $\{Y\}$ is a Markov process and corresponding covariance matrix $[V]$ satisfies [23] the following equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}[V] = [H][V]^{\mathrm{T}} + [V][H]^{\mathrm{T}} + [P], \tag{22}$$

where $[V]^{\mathrm{T}}$ is the transpose of matrix $[V]$.

The elements of matrix $[V]$ and $[P]$ are given by

$$V_{ij} = E[Y_i Y_j], \tag{23}$$

$$P_{ij} = E[W_i Y_j], \tag{24}$$

where Y_i and W_i represents the ith element of the vector $\{Y\}$ and $\{W\}$, respectively. All the elements of matrix $[P]$ will be zero except $P_{(2n+8,\ 2n+8)} = 2\pi S_0$.

The augmented system matrix, $[H]$ is time dependent through the introduction of modulating function, $A(t)$. The non-stationary response of the system (i.e. $[V]$ matrix) is obtained by solving the moment Eq. (22) using step-by-step method. The fourth-order Runge–Kutta method is employed for the present study. It is to be noted that the nonlinear phenomenon of the LRB still exists due to dependence of equivalent constant C_j and K_j on the elements of the $[V]$ matrix (refer Eqs. (17) and (18)). However, this is taken care by modifying the C_j and K_j in each time step depending upon the response of the system.

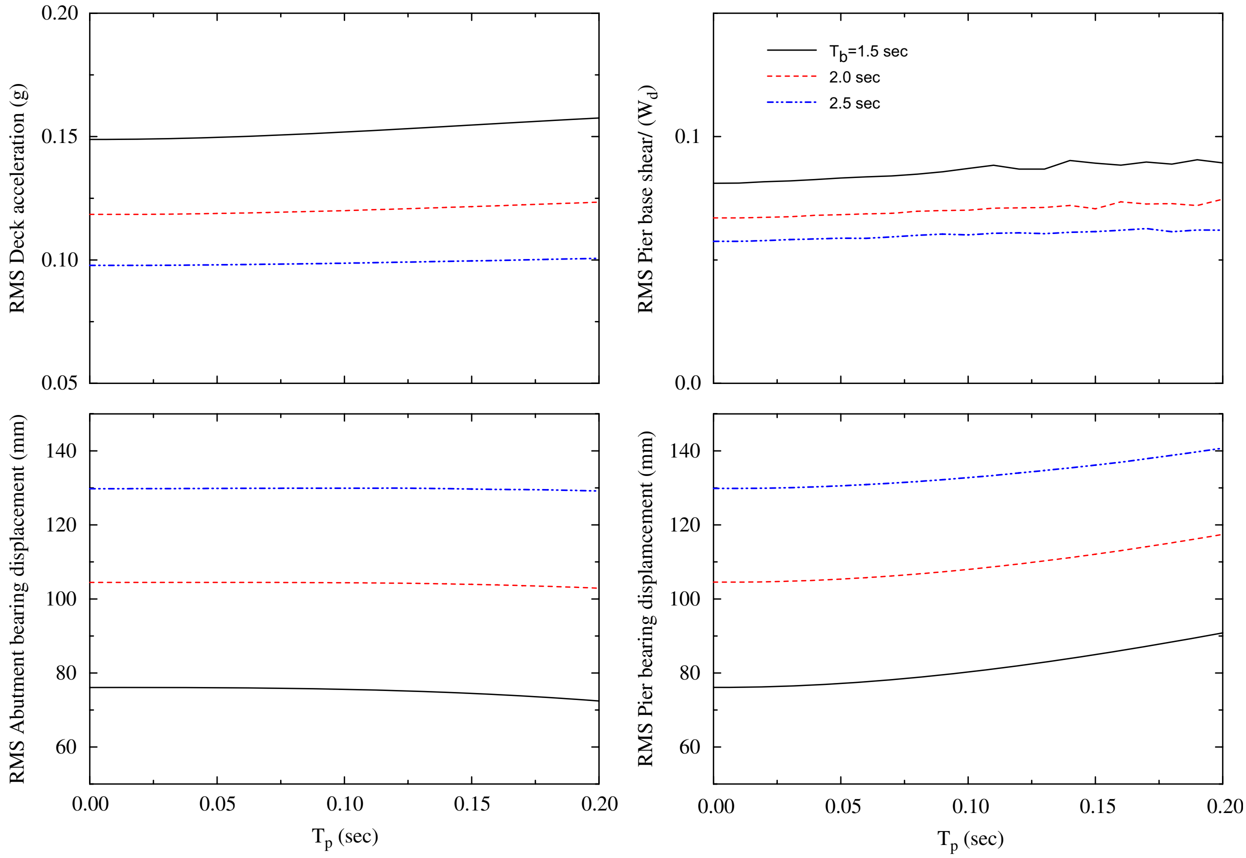

Fig. 3. Influence of the pier flexibility on the response of isolated bridge system ($\mu = 0.15$, $\xi_b = 0.1$, $F_0 = 0.05$, $A(t) = 1$ $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m^2/s^3}$).

6. Numerical study

With the assumed values of parameters, the model of seismically isolated bridge considered in the present study requires the specifications of parameters namely (i) time period of pier (T_p), (ii) ratio of pier mass of pier to deck (μ), (iii) isolation period (T_b), (iv) bearing damping ratio (ξ_b) and (v) normalized yield strength of bearing (F_0). The excitation requires the values of the parameters ω_g and S_0. The non-stationary response of the system is obtained by solving Eq. (22). The response quantities of interest are the root mean square (rms) absolute deck acceleration, rms pier base shear and rms abutment and pier bearing displacements. The absolute acceleration of the deck and the pier base shear are directly proportional to the forces exerted in the bridge system due to earthquake ground motion. On the other hand, the relative displacements of the bearings are crucial from the design point of view of isolation system and expansion joints.

The response of the system is investigated for two types of modulating functions expressed by Eq. (11) namely (i) $t_1 = 2.5$ s, $t_2 = 7.5$ s and $c = 0.5\,\mathrm{s}^{-1}$ and (ii) $t_1 = 2.5$ s, $t_2 = 12.5$ s and $c = 0.5\,\mathrm{s}^{-1}$. These modulating functions are referred as type-I and -II, which has the strong motion duration, T_0 as 5 and 10 s, respectively. In addition, the response is also investigated for $A(t) = 1$ for all values of time t (this corresponds to a stationary earthquake ground motion) in order to distinguish between the stationary and non-stationary response of the seismically isolated bridge system.

In Fig. 2, time variation of the rms deck acceleration, pier base shear and bearing displacements is shown for different types of modulating functions with $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m}^2/\mathrm{s}^3$. The response is shown for the system parameters: $T_p = 0.05$ s, $\mu = 0.15$, $T_b = 2$ s, $\xi_b = 0.1$ and $F_0 = 0.05$ which corresponds to the model

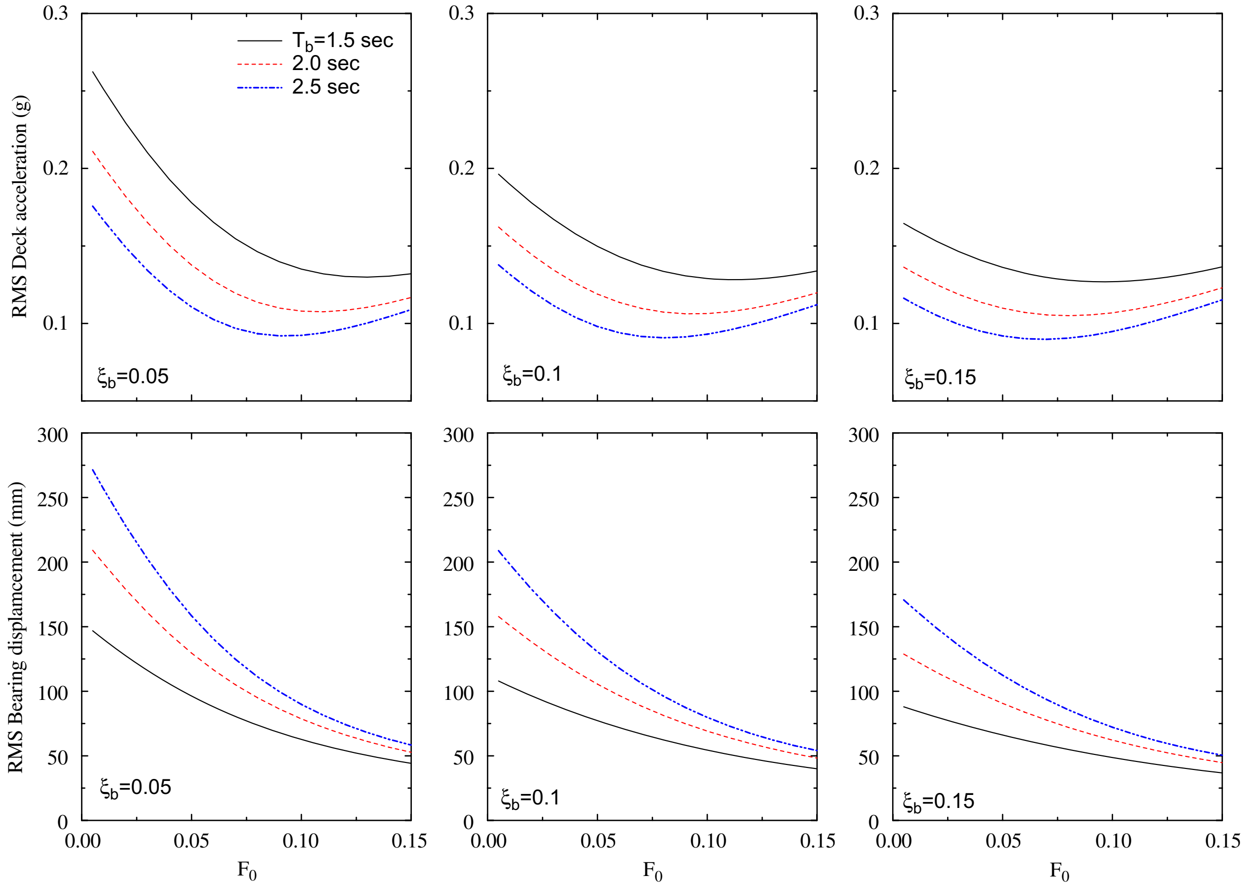

Fig. 4. Influence of the yield strength of LRB on the response of isolated bridge system ($T_p = 0.05$ s, $\mu = 0.15$, $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m}^2/\mathrm{s}^3$).

of the bridge considered in Ref. [10]. It is observed from the figure that the stationary response is achieved in a very short time (i.e. about 2.5 s). In addition, the peak rms response under non-stationary ground motion is the same as that of the stationary response. This happens mainly due to large damping in the isolated bridge system. Thus, the stochastic earthquake response of the isolated bridges can be obtained by considering the stationary model of earthquake ground motion with appropriate frequency variation of PSDF and intensity. In view of this fact, all the subsequent numerical results of the isolated bridge system are presented for stationary condition, i.e. $A(t) = 1$ only.

Fig. 3 shows effect of variation of flexibility of the bridge pier on the stationary rms response of the bridge The pier time period is varied from 0 to 0.2 s considering three values of isolation period (i.e. $T_b = 1.5$, 2 and 2.5 s) with $\mu = 0.15$, $\xi_b = 0.1$, $F_0 = 0.05$, $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m^2/s^3}$. The response for $T_p = 0$ represents the corresponding response of the isolated bridge system with rigid pier condition (i.e. idealized as a single-degree-of-freedom system). The figure indicates that the rms response of the isolated bridge system remains almost constant with the increase of the time period or flexibility of the pier. This implies that the flexibility of the pier does not have significant influence on the response of isolated bridge. This is expected due to the fact that in the isolated bridge system, the flexibility is mainly concentrated in the isolation system and the piers behave as a rigid body. Further, as expected the rms deck acceleration and pier base shear decreases with the increase in the period of isolation. On the other hand, the bearing displacements increase with the increase in isolation period. Thus, the earthquake forces transmitted to the bridge system can be reduced at the expense of increasing relative displacement of the bearings. However, the relative displacement of the LRB has

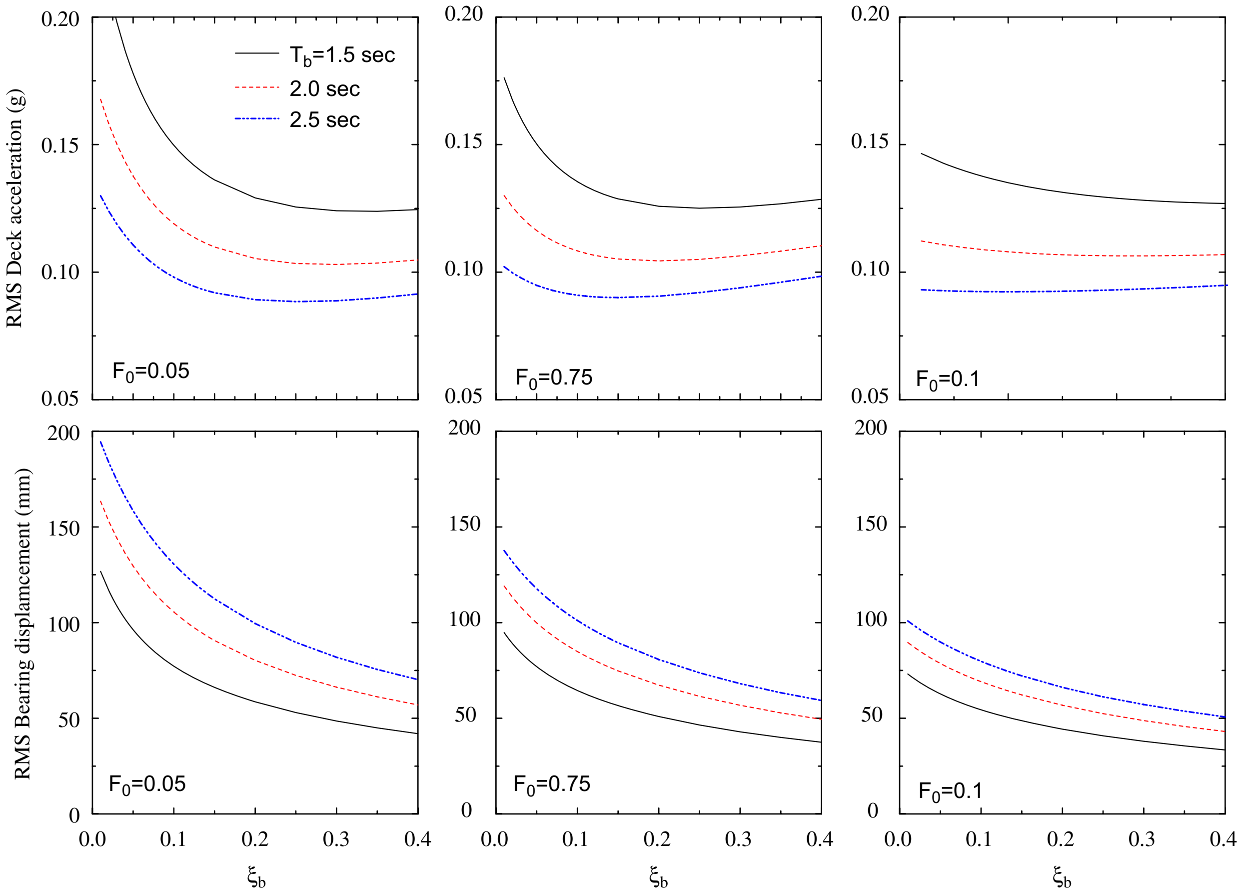

Fig. 5. Influence of the viscous damping of LRB on the response of isolated bridge system ($T_p = 0.05$ s, $\mu = 0.15$, $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m^2/s^3}$).

a practical limitation. Therefore, in designing the isolation system a compromise shall be made between transmitted earthquake forces and relative bearing displacements.

Fig. 4 shows the effects of yield strength of LRB on the rms deck acceleration and bearing displacement for different values of isolation periods and bearing damping ratios. It is observed that with the increase in the yield strength of LRB, the rms deck acceleration reduces first and attains a minimum value and then it increases with the increase of yield strength. This indicates that there exists optimum yield strength of the LRB system for which the deck acceleration attains the minimum value. The comparison of the optimum yield strength for different isolation periods indicates that the optimum yield strength of LRB decreases with the increase of isolation period. Further, the optimum yield strength of LRB also decreases with the increase of the viscous damping of bearing. This is due to the fact that the optimum total damping (due to viscous and hysteretic due to lead core) for a given system is constant. Therefore, for a system with higher viscous damping, there will be less requirement of damping due to lead-plug; as a result, the optimum yield strength is reduced. In Fig. 5, the response of bridge is plotted against the damping ratio of LRB. The effects of the damping ratio on the bridge response are similar to that observed for yield strength. Thus, there exists a combination of the yield strength and viscous damping for which the deck acceleration of a given bridge system and excitation attains the minimum value.

In order to study the reasons for optimum hysteretic and viscous damping of the LRB, the PSDF of the deck acceleration and bearing displacement is plotted in Fig. 6 for different combinations of yield strength and damping ratio of the bearing. It is seen from the figure that at resonance both deck acceleration and bearing displacement are suppressed. However, the PSDF of the absolute deck acceleration increases with the increase of the yield strength or viscous damping of the LRB for higher frequencies (i.e. $\omega > \sqrt{2}\omega_b$). Thus, the existence

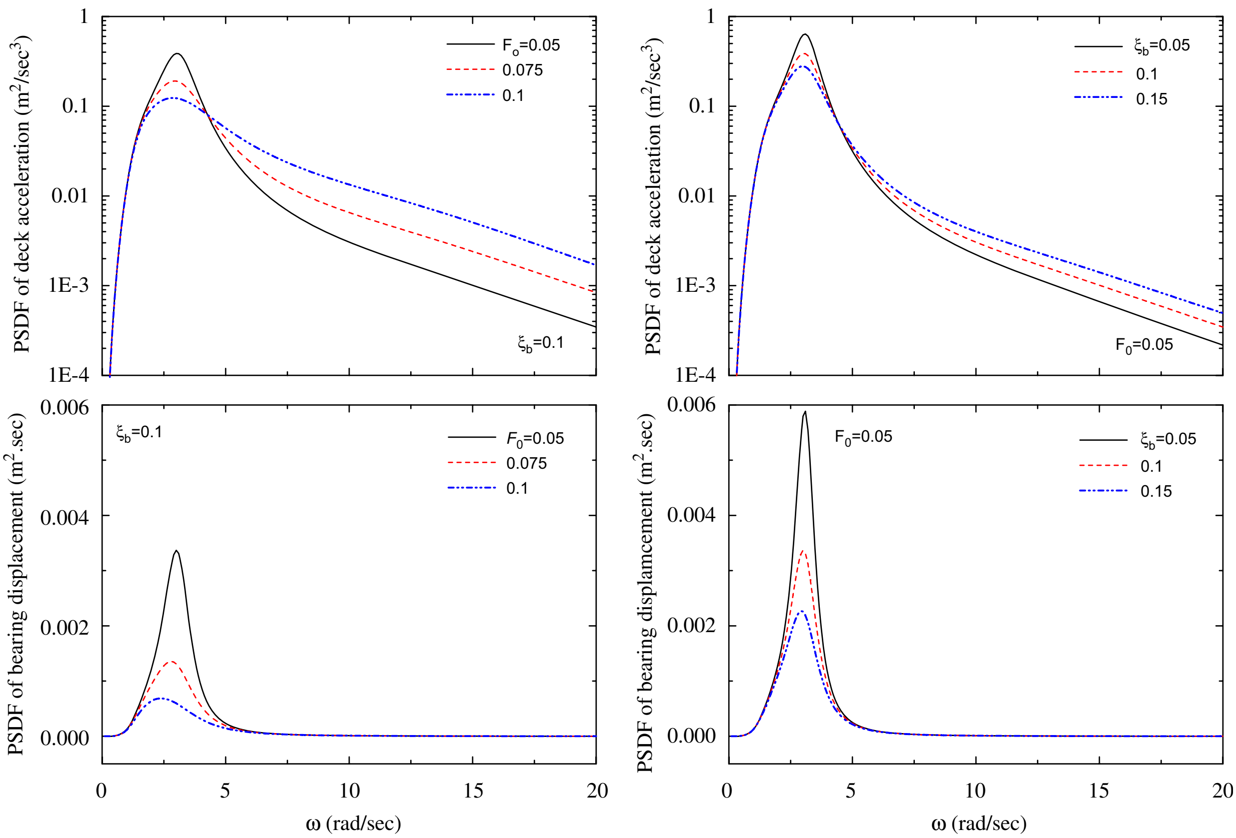

Fig. 6. Variation of the PSDF of the deck acceleration and bearing displacement ($T_p = 0.05$ s, $\mu = 0.15$, $T_b = 2$ s, $\omega_g = 5\pi$ rad/s and $S_0 = 0.05\,\mathrm{m^2/s^3}$).

Table 1
Optimum yield strength of the LRB and response for different system parameters ($T_p = 0.05$ s, $\mu = 0.15$ and $\omega_g = 3\pi$ rad/s)

T_b (s)	S_0 (m^2/s^3)	$\xi_b = 0$			$\xi_b = 0.05$			$\xi_b = 0.1$		
		F_0^{opt}	$\sigma_{\ddot{u}_a}$ (g)	σ_{u_d} (mm)	F_0^{opt}	$\sigma_{\ddot{u}_a}$ (g)	σ_{u_d} (mm)	F_0^{opt}	$\sigma_{\ddot{u}_a}$ (g)	σ_{u_d} (mm)
	0.005	0.05	0.044	16.83	0.045	0.044	16.52	0.039	0.043	16.63
	0.01	0.071	0.063	23.69	0.063	0.062	23.59	0.056	0.061	23.24
1.5	0.025	0.112	0.099	37.56	0.1	0.097	37.16	0.088	0.096	36.93
	0.05	0.156	0.14	54.09	0.139	0.138	53.44	0.122	0.136	53.04
	0.075	0.189	0.171	67.1	0.168	0.169	66.28	0.147	0.167	65.78
	0.1	0.217	0.198	78	0.192	0.195	77.29	0.169	0.192	76.22
	0.005	0.041	0.037	25.51	0.036	0.036	25.41	0.032	0.036	24.83
	0.01	0.058	0.052	36.06	0.051	0.052	35.88	0.045	0.051	35.25
2.0	0.025	0.092	0.083	56.82	0.081	0.082	56.51	0.071	0.081	55.83
	0.05	0.13	0.117	80.43	0.115	0.115	79.64	0.101	0.114	78.62
	0.075	0.159	0.143	98.65	0.141	0.141	97.45	0.123	0.139	96.68
	0.1	0.183	0.166	114.32	0.162	0.163	113.03	0.142	0.161	111.66
	0.005	0.036	0.032	34.01	0.031	0.032	34.49	0.027	0.031	34.08
	0.01	0.05	0.045	49.07	0.044	0.045	48.62	0.039	0.044	47.46
2.5	0.025	0.08	0.072	76.58	0.07	0.071	76.46	0.061	0.07	75.64
	0.05	0.113	0.102	108.45	0.099	0.1	108.12	0.087	0.099	106.32
	0.075	0.138	0.124	133.24	0.122	0.123	131.68	0.106	0.121	130.7
	0.1	0.159	0.144	154.23	0.141	0.141	151.94	0.123	0.14	150.39

Table 2
Optimum yield strength of the LRB and response for different system parameters ($T_p = 0.05$ s, $\mu = 0.15$ and $\omega_g = 5\pi$ rad/s)

T_b (s)	S_0 (m^2/s^3)	$\xi_b = 0$			$\xi_b = 0.05$			$\xi_b = 0.1$		
		F_0^{opt}	$\sigma_{\ddot{u}_a}$ (g)	σ_{u_d} (mm)	F_0^{opt}	$\sigma_{\ddot{u}_a}$ (g)	σ_{u_d} (mm)	F_0^{opt}	$\sigma_{\ddot{u}_a}$ (g)	σ_{u_d} (mm)
1.5	0.005	0.046	0.042	16.29	0.041	0.041	15.97	0.035	0.041	16.06
	0.01	0.065	0.059	23.05	0.058	0.058	22.58	0.05	0.057	22.55
	0.025	0.103	0.093	36.36	0.091	0.092	35.95	0.079	0.091	35.68
	0.05	0.146	0.132	51.29	0.129	0.13	50.73	0.112	0.128	50.37
	0.075	0.179	0.161	62.75	0.158	0.159	62.13	0.137	0.157	61.75
	0.1	0.207	0.186	72.34	0.182	0.184	71.89	0.159	0.181	71.04
	0.005	0.039	0.035	23.22	0.034	0.034	23.27	0.03	0.034	22.78
	0.01	0.055	0.049	32.94	0.048	0.048	32.96	0.042	0.047	32.46
2.0	0.025	0.087	0.077	52.06	0.076	0.076	52.04	0.066	0.075	51.56
	0.05	0.122	0.109	74.35	0.108	0.108	73.27	0.094	0.106	72.53
	0.075	0.15	0.134	90.65	0.132	0.132	89.91	0.115	0.13	88.9
	0.1	0.173	0.154	104.82	0.153	0.152	103.45	0.133	0.15	102.53
	0.005	0.034	0.03	30.25	0.029	0.029	30.98	0.026	0.029	29.79
	0.01	0.047	0.042	43.99	0.042	0.041	42.8	0.036	0.041	42.85
2.5	0.025	0.075	0.066	68.77	0.066	0.065	68.09	0.06	0.064	64.95
	0.05	0.106	0.093	97.33	0.093	0.092	96.63	0.081	0.091	95.34
	0.075	0.13	0.114	119	0.114	0.113	118.25	0.099	0.111	116.95
	0.1	0.15	0.132	137.54	0.132	0.13	136.17	0.114	0.128	135.34

of the optimum yield strength or viscous damping of LRB for minimum absolute deck acceleration is justified and occurring due to high-frequency components of input earthquake motion.

It is observed in Fig. 4 that for a given bridge structural system and specific excitation there exist an optimum yield strength of LRB which produces a minimum rms absolute acceleration of the deck. It will be interesting to study the variation of the optimum yield strength of LRB (denoted by F_0^{opt} and the corresponding rms absolute deck acceleration ($\sigma_{\ddot{u}_a}$) and bearing displacement σ_{u_d} under the important system parameters such as T_b, ξ_b, ω_g and S_0. Note that the criterion selected here for the optimality is the minimization of deck absolute acceleration with unlimited bearing displacement. Table 1 shows the effects of T_b, ξ_b and S_0 on the optimum yield strength of LRB and corresponding rms deck acceleration and base displacement for $T_p = 0.05\,\text{s}$, $\mu = 0.15$ and $\omega_g = 3\pi\,\text{rad/s}$. It is seen from the table that the F_0^{opt} decreases with the increase in the isolation period. On the other hand, the corresponding rms base displacement at F_0^{opt} increases with the increase of the isolation period. This is due to fact that increases in the isolation period increases the flexibility in the system resulting in more displacements. Thus, it is concluded that increase in the isolation period decreases the optimum yield strength of LRB. Further, the optimum yield strength as well as the deck acceleration and bearing displacement increases with the increase of the intensity of earthquake excitation. The dependence of optimum yield strength of LRB on the intensity of earthquake excitation is essentially due to nonlinear force-deformation behavior of the LRB. Thus, the optimum yield strength of LRB depends upon the earthquake intensity; it increases with the increase of the intensity. Similar effects of T_b, ξ_b and S_0 on the F_0^{opt} and corresponding response are depicted in Table 2 showing the results for $\omega_g = 5\pi\,\text{rad/s}$.

7. Closed-form expressions for optimum parameters

Fig. 3 had indicated that the flexibility of the pier does not have noticeable effects on the response of the isolated bridge. As a result, consider an idealized model of the isolated bridge under rigid pier condition as shown in Fig. 7(a) for finding the approximate closed-form expressions for the optimum yield strength of LRB and corresponding response. The idealized model is a single-degree-of-freedom system with mass as that of bridge deck, m_d supported on the LRB characterized by linear stiffness constant, $\sum \alpha k_\text{b}$, damping constant, $\sum c_b$ and hysteretic damping component arising due to lead plug, $\sum(1-\alpha)F_y$. To simplify further, the hysteretic damping of lead plug is replaced by equivalent friction type damping as shown in Fig. 7(b). The limiting friction force of the equivalent device is considered as $a_0 \sum F_y$ (where a_0 is the normalizing constant). Assuming that the equivalent friction device remains in the sliding phase during the earthquake excitation, the governing equation of motion of the model in Fig. 7(b) is expressed as

$$\ddot{u}_d + 2\xi_b \omega_b \dot{u}_d + \omega_b^2 u_d + a_0 F_0 g \,\text{sgn}(\dot{u}_d) = -\ddot{u}_g, \tag{25}$$

where u_d is the displacement of the bridge deck relative to the ground; and sgn denotes the signum function.

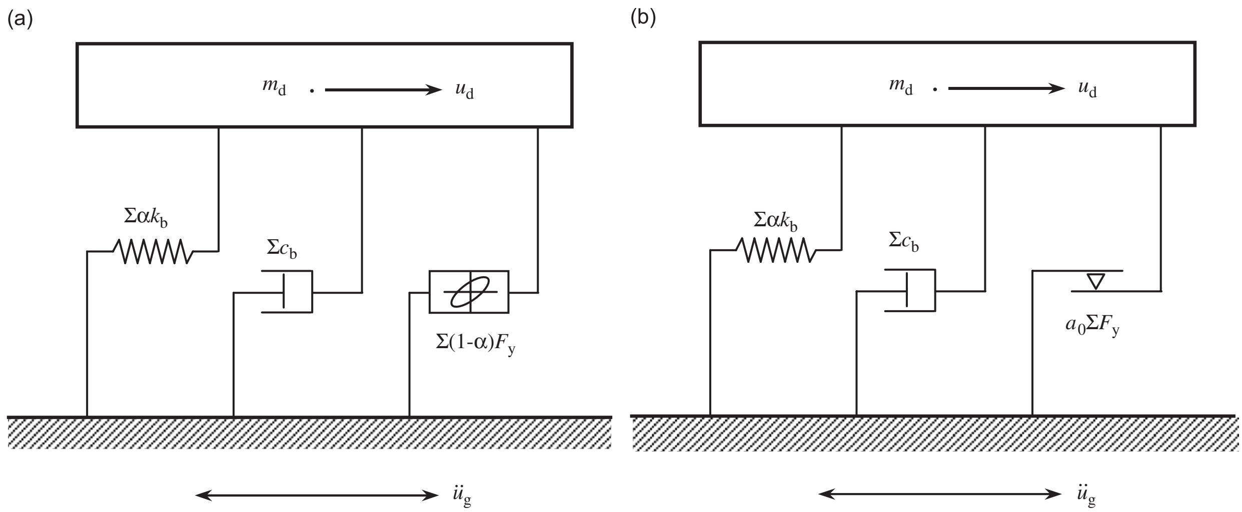

Fig. 7. Simple model of bridge isolated by the LRB.

The equation of motion of an idealized bridge model expressed in Eq. (25) is nonlinear and the corresponding equivalent linearized form is given by

$$\ddot{u}_d + 2\xi_e\omega_b\dot{u}_d + \omega_b^2 u_d = -\ddot{u}_g, \tag{26}$$

where ξ_e is an equivalent damping ratio which is obtained by minimizing the mean square of the difference between Eqs. (25) and (26). The equivalent damping ratio [22,24] is expressed as

$$\xi_e = \xi_b + \frac{a_0 F_0 g}{\sqrt{2\pi}\omega_b\sigma_{\ddot{u}_d}}. \tag{27}$$

Let the earthquake ground acceleration, $\ddot{u}_g$ is characterized by constant PSDF of S_0 in place of the filtered white-noise process. This is a specific case of the model considered in the present study when $\omega_f \to 0$ and $\omega_g \to \infty$. The PSDF function of the system response to white-noise ground motion can be determined by solving Eq. (26) using standard linear stochastic response analysis procedure. The PSDF of absolute deck acceleration, $\ddot{u}_a$ (i.e. $\ddot{u}_a = \ddot{u}_d + \ddot{u}_g$) is expressed by [23]

$$S_{\ddot{u}_a}(\omega) = S_0\left(\frac{1 + 4\xi_e^2(\omega/\omega_b)^2}{[1-(\omega/\omega_b)^2]^2 + 4\xi_e^2(\omega/\omega_b)^2}\right). \tag{28}$$

The mean square absolute acceleration of the deck is given by the area under the PSDF curve given by Eq. (28), i.e.

$$\sigma_{\ddot{u}_a}^2 = \int_{-\infty}^{\infty} S_{\ddot{u}_a}(\omega)\mathrm{d}\omega. \tag{29}$$

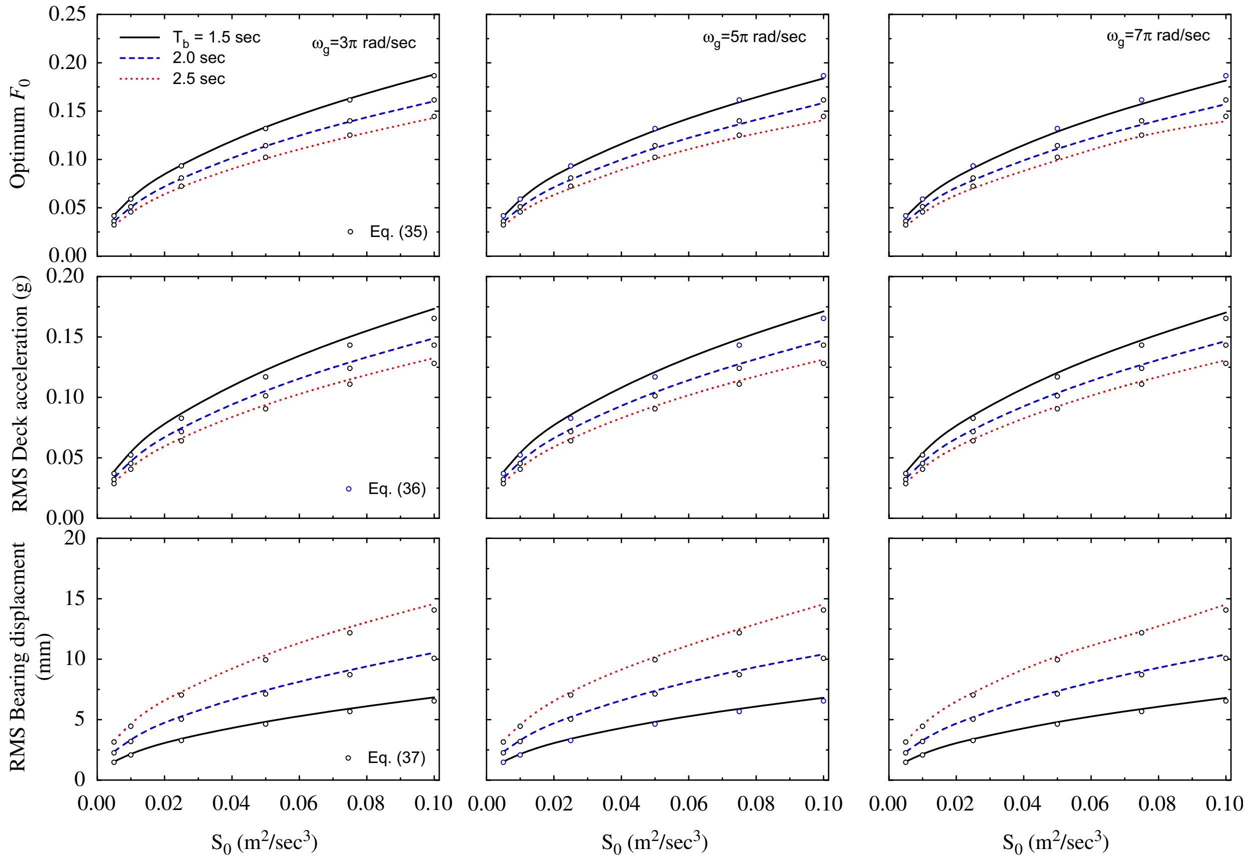

Fig. 8. Comparison of the response of two models for $a_0 = 1/\sqrt{2}$ under different values of T_b, ω_g and S_0 ($\xi_b = 0.05$).

Substituting the expression for $S_{\ddot{u}_a}(\omega)$ from Eq. (28) into the above integral equation and solving, the mean square absolute acceleration of the deck is expressed by

$$\sigma^2_{\ddot{u}_a} = 2\pi S_0 \omega_b \left(\frac{1}{4\xi_e} + \xi_e \right). \tag{30}$$

The value of the optimum equivalent optimum damping ratio for minimum mean square absolute acceleration of the deck can then be obtained by differentiating the response expression with respect to the equivalent damping ration and equating to zero, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}\xi_e}\sigma^2_{\ddot{u}_a} = 2\pi S_0 \omega_b \left(-\frac{1}{4\xi_e^2} + 1 \right) = 0. \tag{31}$$

Solving the above equation yields an optimum equivalent damping, $\xi_e^{\text{opt}} = 0.5$. Substituting this value in Eq. (27), the optimum level of the yield strength of LRB is expressed by

$$F_0^{\text{opt}} = \frac{1}{a_0 g}\left(\frac{1}{2} - \xi_b \right) \sqrt{2\pi}\omega_b \sigma_{\dot{u}_d}. \tag{32}$$

The corresponding expression for the mean square velocity for $\xi_e^{\text{opt}} = 0.5$ is expressed by

$$\sigma^2_{\dot{u}_d} = \frac{\pi S_0}{\omega_b}. \tag{33}$$

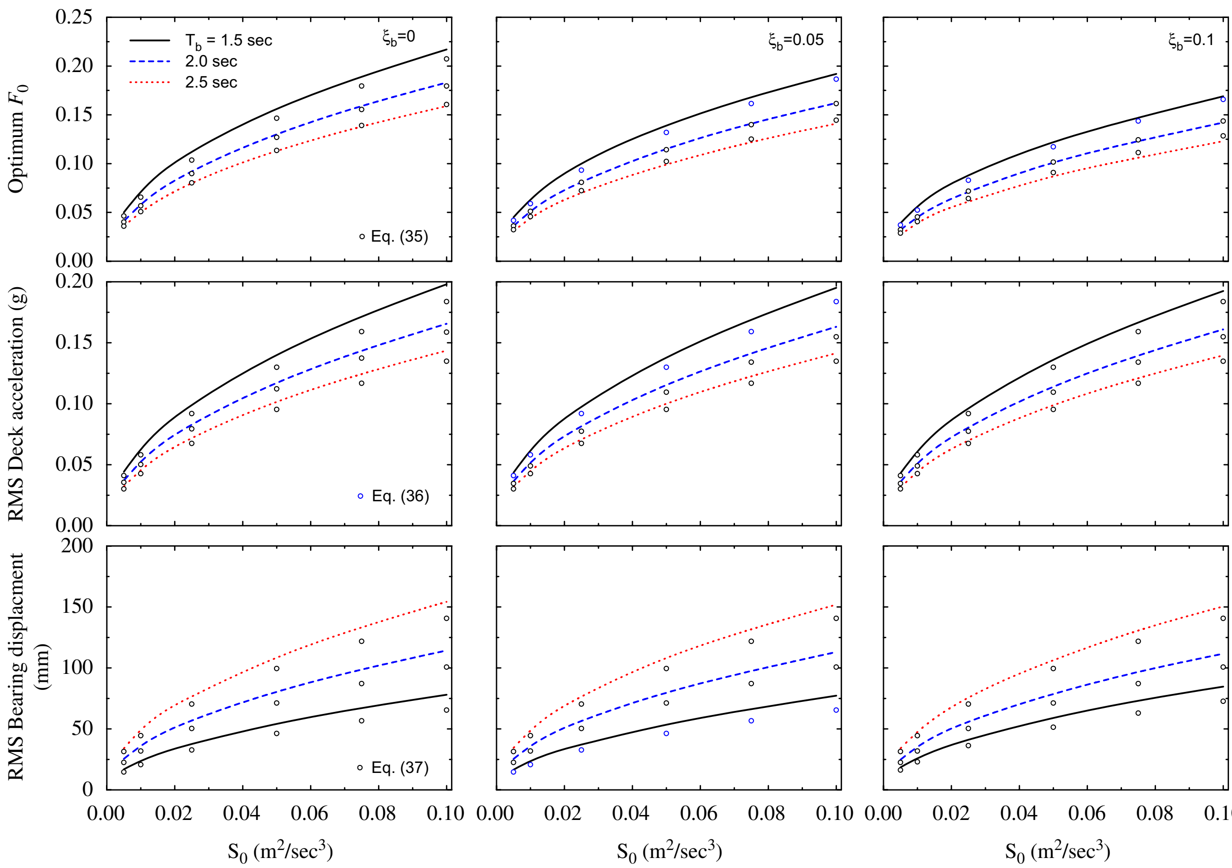

Fig. 9. Comparison of the optimum yield strength of LRB and corresponding response with proposed approximate expressions ($T_p = 0.05$ s, $\mu = 0.15$, and $\omega_g = 3\pi$ rad/s).

Substituting for the rms velocity in Eq. (32), the optimum yield strength of LRB is expressed as

$$F_0^{\text{opt}} = \frac{1}{a_0 g}\left(\frac{1}{2} - \xi_b\right)\pi\sqrt{2\omega_b S_0}. \tag{34}$$

In order to find the normalizing constant, a_0, the optimum yield strength of LRB compared with that obtained numerically considering bridge model shown in Fig. 7(a). It was observed that for $a_0 = 1/\sqrt{2}$, the results of the two models matches perfectly.

Substituting for $a_0 = 1/\sqrt{2}$ in Eq. (34), the expression for the optimum yield strength of LRB can be taken as

$$F_0^{\text{opt}} = \frac{1}{g}\left(\frac{1}{2} - \xi_b\right)2\pi\sqrt{\omega_b S_0}. \tag{35}$$

The corresponding expression for the mean square absolute deck acceleration and relative bearing displacement at optimum yield strength (i.e. $\xi_e^{\text{opt}} = 0.5$) are expressed by

$$\sigma_{\ddot{u}_a}^2 = 2\pi S_0 \omega_b, \tag{36}$$

$$\sigma_{u_d}^2 = \frac{\pi S_0}{\omega_b^3}. \tag{37}$$

In Fig. 8(a) comparison of the response of two models shown in Figs. 7(a) and (b) is made by considering $a_0 = 1/\sqrt{2}$ for different values of T_b, ω_g and S_0. The figure clearly indicates that when the normalizing

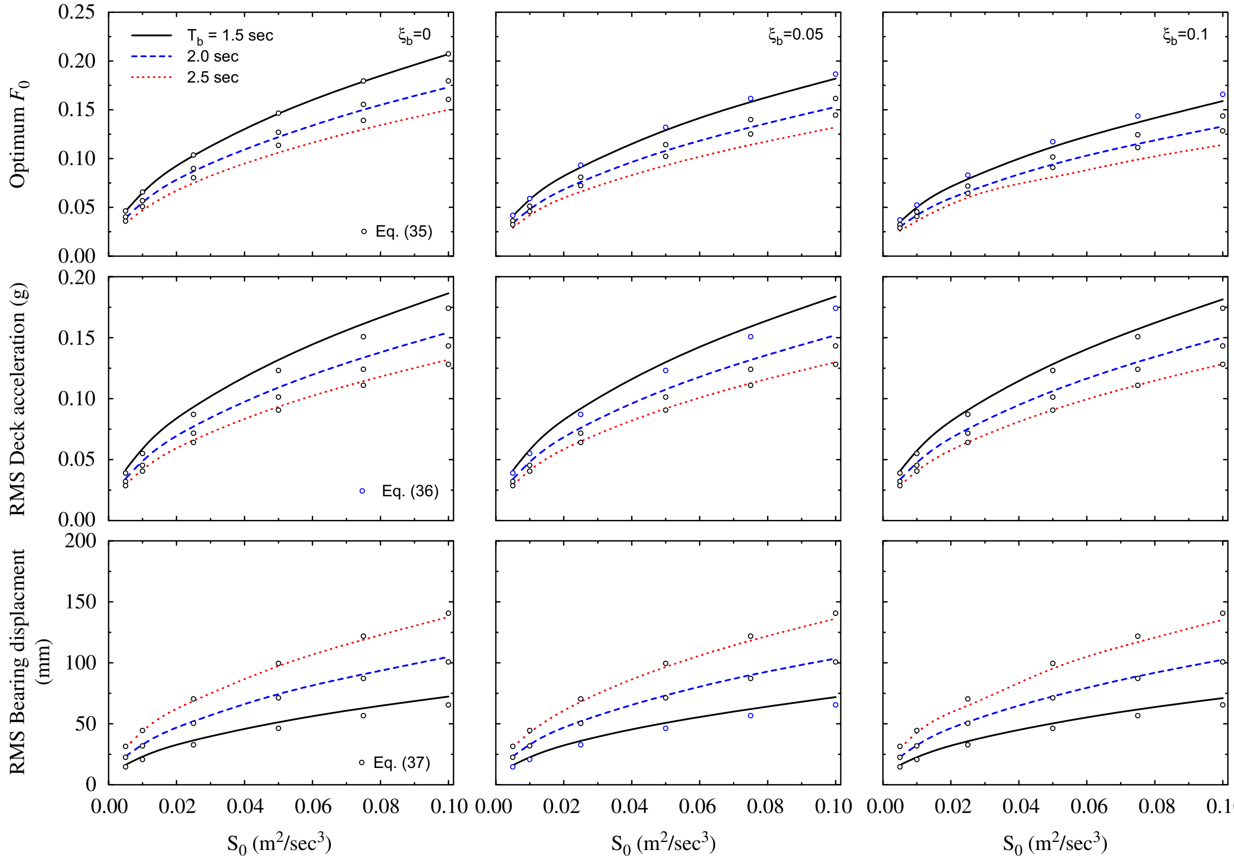

Fig. 10. Comparison of the optimum yield strength of LRB and corresponding response with proposed approximate expressions ($T_p = 0.05$ s, $\mu = 0.15$, and $\omega_g = 5\pi$ rad/s).

constant, $a_0 = 1/\sqrt{2}$ the results of the two models matches perfectly. Thus, Eqs. (35)–(37) provide the approximate expressions for the optimum yield level of bearing and corresponding mean square absolute deck acceleration and relative bearing displacement, respectively. Further, Eq. (35) indicates that the optimum yield strength of LRB increases with the increase of both isolation frequency, ω_b and intensity of earthquake excitation, S_0 confirming the trends of the results shown in Tables 1 and 2.

A comparison of the optimum yield strength of LRB and corresponding response obtained numerical searching technique and proposed approximate expressions (refer Eqs. (35)–(37)) are shown in Figs. 9 and 10 for $\omega_g = 3\pi$ and 5π rad/s, respectively. These figures indicate that there is a good agreement between the actual numerical results obtained in Tables 1 and 2 and that from the proposed closed-form expressions. Thus, the proposed expressions for the optimum yield strength of LRB and corresponding response can be effectively used for preliminary optimum design of bearings for seismic isolation of bridges.

8. Conclusions

Stochastic response of continuous span bridges isolated by the LRB under non-stationary earthquake ground motion is investigated. Method of time dependent equivalent linearization is used to obtain the peak stochastic response of the system. The response of the system is analyzed for the optimum yield strength of LRB. The criterion selected for the optimality is the minimization of the rms absolute acceleration of deck. The optimum yield strength of LRB is investigated under important parametric variations such as: the damping ratio and period of LRB and the pre-dominant frequency and intensity of the earthquake excitation. From the trends of the results of present study, following conclusions may be drawn:

1. The stochastic earthquake response of the isolated bridges can be obtained by considering the stationary model of earthquake ground motion with appropriate frequency variation of PSDF function and intensity.
2. The flexibility of the pier does not have significant effects on the stochastic response of the isolated bridge. Thus, the response of the isolated bridge can be obtained by considering both deck as well as piers as a rigid body.
3. For a given isolated bridge structural system there exist an optimum yield strength of the LRB for which the absolute acceleration of the deck attains a minimum value. However, the bearing displacement goes on decreasing with the increase of bearing yield strength.
4. The optimum yield strength of LRB decreases with the increase of its viscous damping and flexibility.
5. The optimum yield strength of LRB is dependent upon the intensity of earthquake excitation. It increases with the increase of the intensity of earthquake motion.
6. The proposed expressions for the optimum yield strength of LRB and corresponding response had good agreement with the actual results obtained for the isolated bridge model. These expressions can be effectively used for preliminary optimum design of bearings for seismic isolation of bridges.

Appendix A. Details of $[H]$ matrix

$$
[H] = \begin{bmatrix}
0 & \{0\}^{\mathrm{T}} & 1 & \{0\}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\{0\} & [0] & \{0\} & [I] & \{0\} & \{0\} & \{0\} & \{0\} & \{0\} & \{0\} \\
-2\alpha k_b/m_d & \alpha k_b\{\psi\}^{\mathrm{T}} & -2c_b/m_d & c_b\{\psi\}^{\mathrm{T}} & -\dfrac{(1-\alpha)F_y}{m_d} & -\dfrac{(1-\alpha)F_y}{m_d} & -\omega_f^2 A(t) & -2\xi_f\omega_f A(t) & -\omega_g^2 A(t) & -2\xi_g\omega_g A(t) \\
\dfrac{\alpha k_b\{\psi\}}{[m_p]} & -\dfrac{[k_p]-\alpha k_b\{\psi\}\{\psi\}^{\mathrm{T}}}{[m_p]} & \dfrac{c_b\{\psi\}}{[m_p]} & -\dfrac{[c_p]-c_b\{\psi\}\{\psi\}^{\mathrm{T}}}{[m_p]} & \{0\} & \dfrac{\{\psi\}(1-\alpha)F_y}{[m_p]} & -\{1\}\omega_f^2 A(t) & -\{1\}2\xi_f\omega_f A(t) & -\{1\}\omega_g^2 A(t) & -\{1\}2\xi_g\omega_g A(t) \\
0 & \{0\}^{\mathrm{T}} & -C_a/q & \{0\}^{\mathrm{T}} & -K_a/q & 0 & 0 & 0 & 0 & 0 \\
0 & \{0\}^{\mathrm{T}} & -C_p/q & C_a/q\{\psi\}^{\mathrm{T}} & 0 & -K_p/q & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\omega_f^2 & -2\xi_f\omega_f & -\omega_g^2 & -2\xi_g\omega_g \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\omega_g^2 & -2\xi_g\omega_g
\end{bmatrix}
$$

where $\{0\}$ denotes the null vector of size $n \times 1$; $[0]$ and $[I]$ denotes the null and identity matrix, respectively, of size $n \times n$; and the $[m_p]$ in the denominator indicates the pre-multiplication to the numerator quantity by $[m_p]^{-1}$.

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